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THE
DESCRIPTION AND USE 1-4.
OF
POCKET CASES
OF
MATHEMATICAL,
OR
DRAWING INSTRUMENTS:

CONTAINING, PARTICULARLY,

A familiar Explanation of the Use of the PROTRACTOR, PLAIN SCALE, SECTOR, GUNTER'S SCALES, MARQUIO'S PARALLEL SCALES, and the PROPORTIONAL COMPASSES; with several Examples in TRIGONOMETRY, ARITHMETIC, &c.

Together with Plain Instructions for making the several Kinds of
SUN DIALS.

ILLUSTRATED BY COPPER-PLATES.

By N. MEREDITH,

OPTICAL AND MATHEMATICAL INSTRUMENT MAKER TO
HIS ROYAL HIGHNESS THE DUKE OF YORK.

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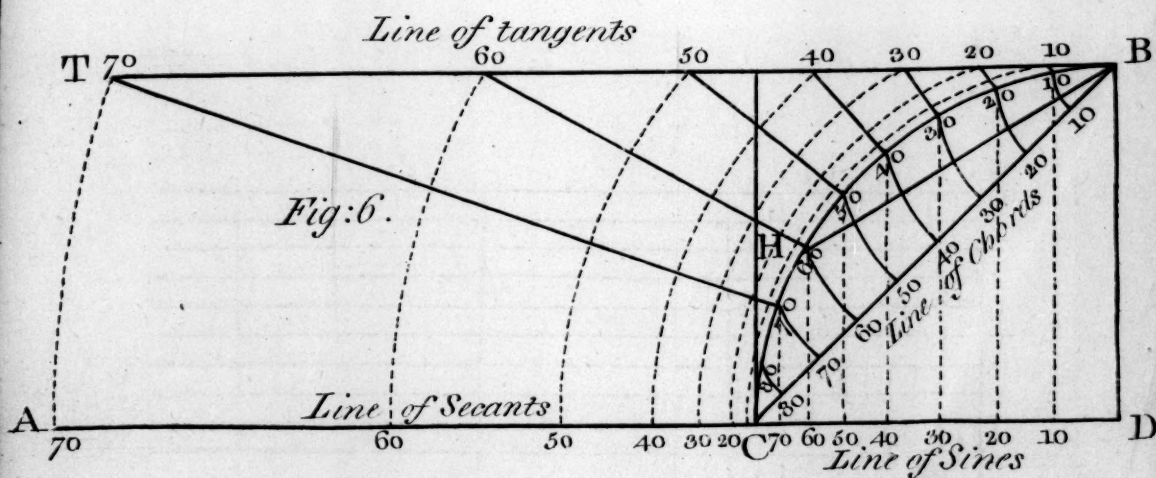
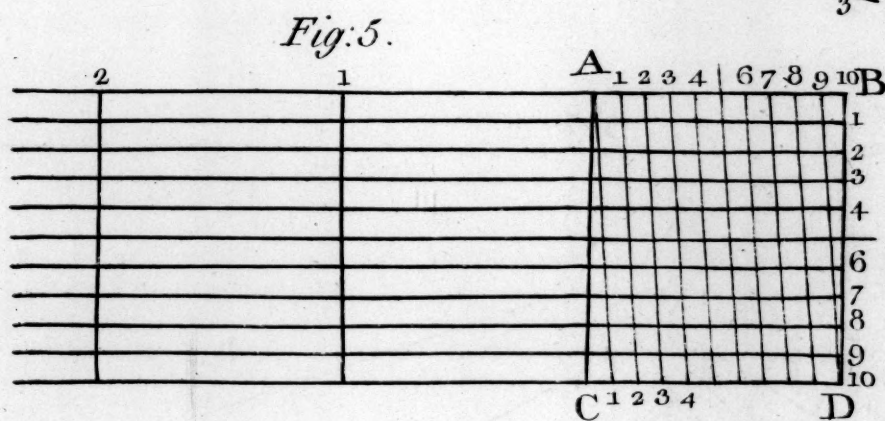
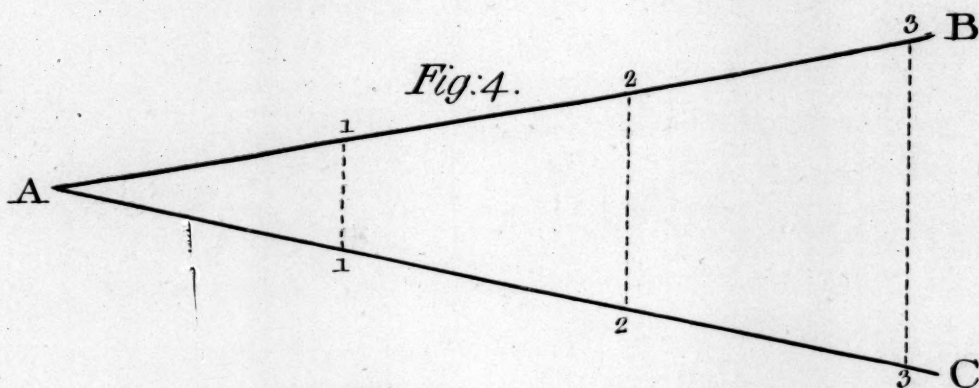
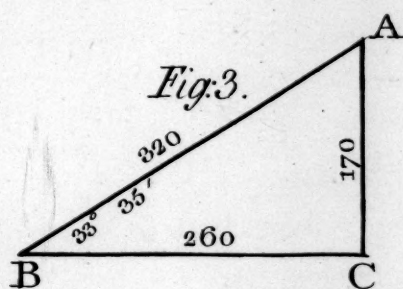
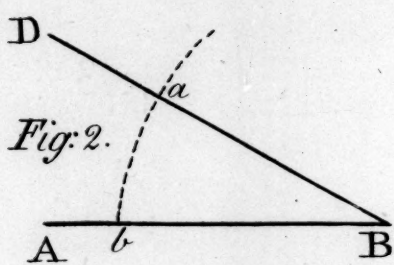
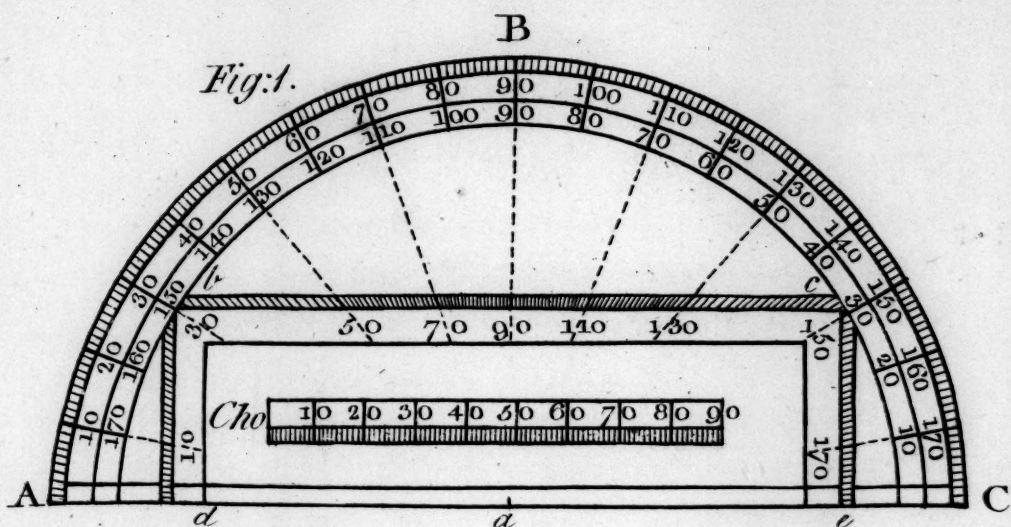
P R E F A C E.

CONSIDERING the number of Ladies and Gentlemen, Artists of various Descriptions, Youth at School, &c. who purchase Cases of Drawing Instruments, for purposes of utility or amusement, it is somewhat extraordinary that there should be no book extant, from which a learner of common capacity may, without much labour, collect the use of the several Instruments which those Cases usually contain. It is a circumstance well known to every shop-keeper, that many persons are not even acquainted with the several uses of the different Compasses and their Appendages; fewer still know the uses of the Plain Scale or Protractor; and as to the Sector, not one in a thousand knows any thing about it, though it is found in every best Case of Instruments, and several of the operations performed with it are extremely easy, and generally useful. Mr. Robertson has, indeed, given an ample description of the several parts of the Case, but his explanations of their uses are far from being so easy as might be wished; and he has entirely omitted the use of the dialling lines on the Sector, though he has employed above eighty pages in describing, &c. the gunner's Calipers, which are very seldom put into Cases of Instruments, and which scarcely any, except those whose occupation leads to it, will ever concern themselves about. In consequence of this, and other things, his book, though to some extremely useful, is by no means generally

generally so, not to mention that, being now out of print, it is not easily procured. It is true, a small tract, written by the late Mr. Martin (and which has been lately republished), is in general much plainer and more easily understood; but, though it is said to contain a description of all the lines on the Sector, yet, as four of those lines which are always found on the best Sectors are (together with several other things of some importance) entirely omitted, and as some parts of it are inapplicable to the Instruments as they are now made, and therefore must tend rather to perplex than instruct; it does not appear, at least to me, to be capable of conveying all the information which even the generality of those who purchase Mathematical Instruments might wish to meet with. Whether the present Publication is better calculated to answer that purpose, the Public, and not its Author, must determine. He can only say, that he has laboured to make it easy to every Reader; and hopes, among other things, the addition of the Dialling Lines will not be unacceptable. Dialling itself, when properly understood, is a very pleasant amusement, and its necessary connexion with some of the least difficult, but not least important, parts of Geography and Astronomy render it still more so.

DESCRIPTION,





DESCRIPTION, &c.

POCKET Cases of Mathematical Instruments admit of a very considerable variety in respect of number, size, and workmanship, according to the fancy or ability of those who purchase them; the most common *size* is either six inches or four and a half (though the former is more frequently used), and what is termed a full or *complete* case contains the following: viz. three pair of *Compasses*, which are useful for different purposes; a *Drawing Pen*, a *Protractor*, a *Sector*, a *Plain Scale*, a *Parallel Ruler*, together with a *Black Lead Pencil*, furnished with a brass flatted top called a *Feeder*; to these are sometimes added a pair of *Proportional Compasses*; the several uses of all which we are now to explain; and first,

OF THE COMPASSES.

THE *Plain Compasses* (or, as they are generally called, the *Dividers*) are those whose points are not made to take out as the points of the *Drawing Compasses* are: the uses of these are so obvious that they require

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very little explanation: it will naturally occur to every one that they may be used in measuring dimensions or distances of every kind, in order to determine their difference or agreement, and likewise in transferring them from one place to another; and though these ends might be answered equally well by the *Drawing Compasses*, with their steel points, yet as measurements are frequently required, while those are in use for other purposes, some time and trouble will be saved by making use of the Dividers, especially as we are frequently desirous of *retaining one* measurement while we take *another*. In the best Cases of Instruments the Dividers or Plain Compasses are so constructed that one of the steel points (besides being moveable at the joint) has also a motion of its own, by means of a screw which presses against a spring connected with the point, by which it may be adjusted to the greatest *nicety* imaginable. When they are so constructed they are denominated *Hair Compasses*, from their being capable of adjustment even to a hair's breadth: these will be found particularly useful in taking distances from the scales, as it is almost impossible to set the others to such a degree of nicety.

The use of the Drawing Compasses is to draw circles and parts of circles, or in general any kind of figure composed of circular lines; and as it will sometimes be required to draw these with ink, and at other times with black lead, &c. therefore one of the steel points of the Compasses is made to take out: this is sometimes effected by loosening a screw
in

in the middle of one of the legs; but in the best sort the moveable pieces are made to slide in and out by means of a springing socket, without the trouble of unscrewing. These Compasses, beside the steel point, are furnished with a *pen* for drawing with ink, and a port crayon to contain a lead pencil, and sometimes with a dotting wheel for describing dotted lines, though this last is not now in much request, principally on account of the difficulty of making them act well, and are therefore seldom found in those Cases of Instruments which are fitted up in the best manner. The two sides of the steel pen may by means of a screw be moved nearer to or farther from each other, that the lines drawn with them may be either stronger or finer, according as they are required: both the pen and pencil points have a joint of their own, so that in every opening of the Compasses they may be set perpendicular to the paper which is necessary for drawing a line well. Besides these two, viz. the Drawing Compasses, and the plain ones, or Dividers, there is a small sort called *Bows*, or *Bow Compasses*, which make a part in every good Case of Instruments; they consist of a steel pen and point of a small length, which are opened with a joint like the other Compasses, but are turned round by a handle or shaft over the joint: they are very useful in describing small circles; which cannot so easily be done with the other Compasses, and they will frequently save the trouble of exchanging the points in the larger Compasses when the pencil point happens to be in

use. These are sometimes, but not very often, made to hold a black lead pencil.

The Drawing Pen, so called, is the same as the pen or ink point which fits into the Compasses, only it is furnished with a stem or handle to hold it by: the use of it is to draw straight lines; it may likewise be used in making figures or letters, but the common writing pen, if at hand, will answer that purpose much better. The handle of the Drawing Pen unscrews in the middle, where there is a steel point, which is useful either for tracing any draft, or for making a fine dot on the paper in order to mark where lines are to terminate, or through which point they are to be drawn.

The uses of the Black Lead Pencil are so well known that nothing need be said on the subject; but it may be necessary to mention that the flat part of the brass top with which it is furnished is used for supplying the pen with ink by dipping it into the ink-stand, and applying the ink thus taken up to the open part of the pen, which by this means is made to hold a greater quantity of ink than could be taken up by dipping the pen itself into the ink-stand, not to mention that when the pen is dipped into the ink, it generally adheres most to the outside, and is more liable to blot the paper in using: on account of this use of the pencil top it is generally called a *feeder*.

The next Instrument we shall mention is the Parallel Ruler, the construction of which is so simple, and its use so obvious, that little need be said about it. It will naturally strike every person who sees it,
 6 that

that as the two rulers of which it is composed are connected by two brass bars of equal length, they must at all distances be parallel to each other, and consequently that nothing more is required in ruling any number of parallel lines than to hold *one* of the bars steady by pressing the fingers on it, while you move the other to the distance required. If it is required to draw one or more parallel lines *under* a given line, the Ruler must be opened before you begin to use it; but, if the lines are to be drawn *over* another line, the Ruler must be shut when you begin to use it.

There is a kind of Parallel Ruler lately invented, known by the name of Marquois Parallel Scales, and sometimes making a part of the larger Cases of Instruments, by which parallel lines, &c. are drawn with great accuracy, a description of which will hereafter be given,

OF THE PROTRACTOR,

THE *Protractor*, so called from its use in *protracting* or *lengthening* angles, or rather the *lines* which form them, is made of different materials and in different forms; the most common are of brass, and their form that of a semi-circle ABC, fig. 1. which form seems the most natural, because on them the divisions are all of the same breadth: it is divided into 180 degrees, and by a double row of figures numbered to the right and left hand of the Protractor, in the middle of the straight bar AC, which forms the diameter, is a fine notch or line *a*, marked on the external edge of the

bar, which is sloped off down to the under side, and is called the *fiducial edge*. The Protractor, in common cases, is about 4 inches in diameter, but it is easy to conceive that the size would not be material if there were no errors in the dividing, nor any likely to happen in the use of it; but as errors in a larger instrument are less in proportion than in smaller ones, the larger are on that account to be preferred. The use of the Protractor is two-fold, viz. to *lay down* any angle, or to *measure* an angle already laid down. If it were required from the point B, fig. 2, to draw a line which should make with the line AB an angle of any given number of degrees, as 30 for instance, lay the fiducial edge of the Protractor exactly even on the line AB, with the mark in its centre on the end B (from whence the other line is to be drawn). Hold the Protractor firmly down on the paper in this situation, and make a dot on the paper exactly where the 30th degree falls; which suppose at *a*; remove the Protractor, and draw the line BD through the point or dot, at *a*, which will make an angle of 30 degrees with the first line AB.

2dly. Suppose the lines AB, BD, already drawn, and you want to know what angle they make with each other, you have only to lay the fiducial edge of the Protractor evenly on one of the lines, with the mark in the middle on the point B, where they meet, and observing at the same time what degree on the Protractor is cut by the other line, you have the quantity of the angle sought; thus if the Protractor be laid on the line AB, as before directed, the *other* line
BD

BD will cut the 30th degree, which shews that they make an angle of 30 degrees with each other. It is now customary in the better sort of Instrument Cases to make the Protractor and Plain Scale but one Instrument, in order to make room for the Parallel Ruler, which is seldom found in the common sorts: it is then generally made of ivory, but sometimes of brass, the Protractor being on one side and the Plain Scale on the other; in this case the Protractor is not in the form of a semi-circle, but in that of a parallelogram or *long square* (as it is generally called), like that represented *b c d e* within the semi-circle. Though this form does not seem so natural, yet its use is the same, and the divisions may be laid on as accurately in this form as in the other, for it is easy to perceive that if from every degree of the semi-circle ABC lines are drawn toward the centre *a*, they will mark the same degrees on the Protractor *b c d e*, by which any angle may be measured or laid down, as well as from the semi-circle itself. In this form of the Protractor the divisions about the ends will be wider than in the brass semi-circular one usually put into common Cases, because they are made longer (being generally 6 inches in length), but they will likewise be *closer* about the middle, and consequently more difficult to use *correctly* in that part, so that after all it may perhaps be questioned whether in this instance utility is not sacrificed to neatness and portability.

OF THE PLAIN SCALE.

THE Instrument called a *Plain Scale* is more properly an *assemblage* of *different scales* laid down on a slip of brass or ivory (most commonly the latter), and contains (besides a Protractor, which in the best Cases is generally placed on one of its sides) as follows:

1st, A Diagonal Scale; 2d, six different Scales of equal parts of an inch; 3d, one or more lines of chords; 4th, a line of six inches subdivided into tenths of an inch; 5th, a line of 50 equal parts. These are differently placed in different Cases of Instruments; in some the two latter with the Diagonal Scale are on one side, and the six scales of equal parts, and a line of chords on the other: but in the best Cases of Instruments the line of 50 equal parts of a foot, and the line of inches, are both omitted to make room for the Protractor, which, together with a line of chords, makes up one side of the Instrument as represented *b c d e*, fig. 1. (leaving out the semi-circle); the other side contains the six scales of equal parts of an inch, with another and smaller line of chords, and the Diagonal Scale.

The uses of the scale of inches, and that of 50-100ths part of a foot which is placed next to it, being the same here as on all other rules, are so easy that they require no explanation; these, as we have already said, are not found on the scales contained in the best Cases of Instruments, nor is their absence at all

all to be lamented, as they are both contained in the lines on the Sector, which is itself contained in every best Case, where only they are wanting on the Plain Scale; and, as they are *twice* as long on the Sector, they are of course more useful.

The six different scales of equal parts of an inch are usually called *Plotting Scales*, from their use to the surveyor and architect in drawing plans, &c. who by the use of these scales can vary the size of his drawing, in the same proportion as they vary from each other: they are usually divided into from 30 to 60 parts of an inch, and the several proportions of the divisions marked at the beginning of the line. There are ten of the small divisions at the beginning of each line, and the remainder of the line is divided into spaces, which are equal to each other, and likewise equal to the space which contains the ten smaller divisions; the larger divisions are numbered 1, 2, 3, &c. and are reckoned as 10, 20, 30, &c. because each is equal to 10 of the small divisions. The use of these scales being very simple, a single example will suffice to explain it (and perhaps even that may be to most unnecessary): suppose, for instance, it were required to draw a line equal in length to 34-40ths of an inch, you have only to take the line which is marked 40 at the beginning, and, placing one foot of the Compasses on the division marked 3 in the same line, extend the other foot of the Compasses over four of the smaller divisions; then, as the points of the Compasses will take in three of the larger divisions, each of which is equal to 10-40ths of an inch, and like-
 wise

wise four of the smaller ones, each of which is 1-40th, it is plain that the points of the Compasses are open to the distance of 34-40ths of an inch, and will consequently give the length of the line required. It has been already observed, that the larger divisions are numbered 1, 2, 3, and so on to 10, which stands for 100, after which the figures begin again; but, as the first time they are reckoned as 10, 20, 30, &c. so the second time they occur they are reckoned as 110, 120, 130, &c.

The Diagonal Scale, which is next to be considered, is a most useful invention; by it any line may be drawn or measured in inches and parts of an inch with great facility, and with extreme accuracy. In order that a proper idea may be formed of the use of this Scale, we must shew the manner in which it is constructed: Let A, B, C, D, fig. 5, be a square inch divided horizontally by parallel lines into ten equal spaces, each of which will of course be equal to the tenth part of an inch; divide the upper and lower lines likewise into ten equal parts marked 1, 2, 3, &c.; then, from A at the beginning of the upper line, draw a line to 1 at the lower line, another from 1 at the top to 2 at the bottom, and so on to the end, and the Diagonal Scale is thus completed. Now as the diagonal lines, in their progress from the top to the bottom, pass over the ten *horizontal lines*, and gain in the *whole* of their passage 1-10th, or 10-100ths of the *whole inch*, so as to be so much more distant from the line AC at the lower end, than they are at the top, and as 1-10th of this is gained at every horizontal line, it follows that the first
diagonal

diagonal line, when it crosses the first *horizontal* line, will be 1-100th of an inch distant from the line AC, at the second horizontal it will be 2-100ths, at the third 3, and so on; in like manner the second diagonal line 1, 2, when it crosses the first horizontal line, will be 1-10th of an inch and 1-100th distant from the line AC, which is equal to 11-100ths of an inch; when it crosses the second horizontal line it will be 12-100ths, and so of all the rest.

In long Diagonal Scales it is usual to divide a *whole* inch, as in the figure; and it is so done here in order to help the eye, and make the nature of its construction as plain as possible: but on the six inch Scales, such as are usually put into Cases of Instruments, instead of an inch being thus divided by diagonal lines, it is usual to put a half inch so divided at one end, and a quarter of an inch at the other; the space between them is divided at the top into spaces of half an inch each, and at the bottom into spaces of a quarter of an inch, the former to be used with the half inch diagonal divisions, and the latter with those of the quarter of an inch: one hundredth part of the former, therefore, gives the 2-100th of an inch, and one of the latter the 4-100th, which is a degree of nicety beyond which nothing of this kind can ever be required.

From what has been said it is apprehended that the use of the Diagonal Scale will be tolerable easy; but, if any difficulty should remain, an example or two will effectually remove it.

Thus, suppose it was required to take off 2 inches and 35-100ths; if the half inch Diagonal Scale is used,
you

you must take double the number of hundredths from the Scale, which is 70, as 70 hundredths on this Scale are only equal to 35-100ths of an inch; now as there are no *odd* hundredths required, take the uppermost line, and, placing one foot of the Compasses at 4, or the fourth principal division at the top (which is 4 half inches from the diagonal divisions), extend the other along the same horizontal line to the 7th diagonal, which will give the length required, the 4 half inches being equal to 2 inches, and the 7-10ths, or 70-100ths, being equal to 35-100ths of an inch.

Again, suppose a line of 3 inches and 43-100ths were required, double the hundredths as before, which in this case will be 86; but, as there are now 6 *odd* hundredths, instead of using the uppermost line as before, you must take the 6th *below* it, and, placing one foot of the Compasses on that line, under the figure 6 (which is 3 inches from the diagonal divisions), extend the other to the 8th diagonal on the same line, which will give the length required, the 8 whole spaces between the diagonal lines being equal to 80-100ths, and the remaining space between the first diagonal and the vertical line AC being equal (as before explained) to 6-100ths.

When the Diagonal Scale at the other end is made use of, as it contains only half the former space, viz. one quarter of an inch, it is obvious that *one* hundredth part of that will be only equal to the 4-100th part of an inch, and so in proportion.

The last line we have to notice on the plain Scale is the line of *chords*, the construction of which will be explained

explained when we come to treat of the Sector; at present we shall confine ourselves to its *use*. There are generally two lines of chords on the best Scales, viz. one at the end of the uppermost line of equal parts, and one on the other side, with the Protractor, as already mentioned; these are taken from circles of different radiuses, as may be known by taking the length of the chord of 60 degrees, that being always the same with the radius, or semi-diameter, of the circle from whence the chords are taken. Their use is the same with that of the Protractor, viz. to lay down or measure angles; and, though the latter has, in some cases, the advantage in point of *expedition*, yet where *accuracy* is required the line of chords is most frequently used.

Suppose it were required to draw a line DB, fig. 2, which should make an angle of 30 degrees with the line AB, having set one foot of the Compasses at the beginning of the line of chords, extend the other to 60 degrees, and, with the same opening of the Compasses, place one foot in B (the point from whence the other line is to be drawn), and with the other describe a portion of a circle, as *ab*; then setting one foot of the Compasses as before at the beginning of the line of chords, extend the other to 30 degrees; then, placing one point at *b*, make a dot where the other point falls on the circle, which suppose at *a*; draw a line from B through this point, and it will make with AB the required angle.

Or, suppose the lines already drawn, and you are to find out what angle they make with each other; take
the

the chord of 60 degrees from either of the lines of chords, as before, between the points of the Compasses, and, placing one foot on B, where the two lines meet (which we now suppose to be already drawn), describe part of a circle as before directed; then taking the arch, or that part of the circle, which is between the two lines, as *a b*, in the Compasses, it will be found to measure 30 degrees on the line of chords, and thus the quantity of the angle is shewn.

OF THE CONSTRUCTION AND USE OF THE SECTOR.

THE Sector is a most valuable instrument; its extensive use in finding out, or laying down, proportions of different kinds has given it, among the French, the title of the Compass of Proportion: it not only contains all the lines usually laid down on the Plain Scales, &c. but the peculiar advantage of its construction is, that, instead of being confined to one particular radius, as on the Plain Scales, the lines of chords, sines, tangents, &c. may be adapted to any radius between the length and breadth of the Sector when opened.

The lines, or scales, which the Sector usually contains are,

1st, Two lines of lines, or equal parts, marked L at the end, and one on each leg of the Sector, reaching from the centre to the ends of the instrument; these are each of them divided into a hundred parts, and marked at every tenth division, 1, 2, 3, &c. to 10; these

hundred parts are likewise generally sub-divided into halves.

2d, Next under these are two lines of secants marked SE. at the ends; these do not reach to the centre, but commence at the 25th degree of the line of equal parts.

3d, Under the secants are two lines of chords reaching from the centre, and marked C. at the ends; these are divided and subdivided like the line of lines, only the divisions are wider, and instead of being numbered 1, 2, 3, they are numbered 10, 20, to 60.

4th, Nearest to the inside of each leg there are two lines of polygons marked POL.; these fall very short of the centre, and are numbered from 4 to 12, the highest number being (contrary to all the other lines) toward the centre of the instrument. These are all contained on one side of the Sector.

5th, On the other there are two lines of sines reaching from the centre, marked S. at the ends, and numbered at every 10th division, 10, 20, &c. to 90; these are likewise usually subdivided into halves as far as to 70 degrees, after which the divisions become too close to admit of it.

6th, Under these on each leg there are two lines of tangents; the lower ones reaching from the centre of the Sector are marked T. at the ends, and numbered every 10th division from 10 to 45, which ends the line; the divisions are likewise subdivided into quarters. The upper tangent lines are marked TA. at the ends; these do not begin at the centre, but at some distance from it, and are numbered from 45
to

to 75; these last are sometimes called lesser tangents, as being the tangents of a lesser circle than those below them. These are all the lines that properly *belong* to the Sector, being all that are to be used Sectorwise: but beside these there are certain lines called *Gunter's* lines, from their being first invented by Mr. *Edmond Gunter*; they consist of a line of *artificial* tangents marked TAN. another of *artificial* sines marked SIN. and one of *artificial* numbers marked NUM.: the last of these is continued the whole length of the Sector when it is *opened* (as it always is when any of these lines are used), but the lines of sines and tangents are shortened by the crossing of the sectoral lines; they are drawn on the same side of the Sector with the natural sines and tangents: on the other side is a line of inches the same as on a common rule, and by it, on the outside edge, is frequently a scale of a foot divided into 100 parts. The best (or what are called full divided) Sectors likewise contain, in addition to all these, a line of inclined meridians, a single line of chords, a line of latitudes, and a line of hours, which are placed on the other side of the Sector near the joint.

We should now proceed to explain the *uses* of these several lines, but as a very imperfect idea will be formed of their *use* unless their *construction* is first understood, we shall now shew their generation *from*, and their relation *to*, the *circle*, from whence they all originate.

From the point D, fig. 6, as a centre, describe the Quadrant, or quarter of a circle, CHB, and divide it into 9 equal parts, numbering them 10, 20, &c. then
will

will each division answer to 10 degrees, and the Quadrant be divided into 90 degrees. If from the point B a right line be drawn to any part of the Quadrant, this line is called the *Chord* of that arch; thus the line BH is the chord of 60 degrees: and if the distance of the several divisions of the Quadrant from the point B be taken with a pair of compasses, and transferred to the said line BC (as in the figure), it will be divided into a *line of chords* answering to every division of the Quadrant. In this manner the lines of chords on the Sector (as well as on the Plain Scale) are formed from a circle, whose radius is equal to the chord of 60 degrees, for if the length of the chord BH be measured, it will be found equal to the radius DB, or DC. Note, the single lines of chords, whether on the Sector or plain scale, are always extended to 90 degrees; but the *double* lines of chords on the Sector are only extended to 60 degrees, the length of the Sector (when shut) being the radius of the circle from whence the chords are taken, as it is likewise of the sines, tangents, &c. The tangent of 45 degrees, the sine of 90, and the chord of 60, being all equal to each other and to the radius, for which reason the chords on the Sector end at 60, the larger tangents at 45, and the sines at 90; all which will be plain from the figure.

From the points 10, 20, 30, &c. on the Quadrant draw right lines parallel to BD, till they touch the line CD, then will CD be divided into a *line of sines*, such as on the Sector. Draw BT perpendicular to BD, and touching the Quadrant at B, and it will

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form

form a *tangent*, so called from its *touching* the circle without cutting it; and if you lay a rule from the centre D, over the several points marked 10, 20, &c. on the Quadrant, and draw right lines till they intersect the line BT, then will BT be a line of tangents: and as the lines so drawn from the centre D till they reach the tangent line are called *secants* of their respective arches; if their lengths from D are transferred by the compasses to the line CA, it will be a line of *secants*.

The lines of lines on the Sector, are formed by dividing the radius into a hundred equal parts.

The lines of *lesser* tangents marked TA. are constructed in the same manner as the larger, only from a smaller circle: (therefore the radius or 45 degrees is much nearer to the centre of the instrument) and beginning at 45 degrees, where the larger end, are continued to 75 degrees or upwards.

We have already observed that the lines are laid on the Sector by pairs, viz. one on each leg; they all issue from the same centre, and every pair of lines make the same angle: and this being the case when the Sector is shut up, it must necessarily be the same in every degree of opening, so that to whatever angle the lines of lines, for instance, are opened, to the same angle are the lines of *sines*, and all the rest opened. It is further necessary to observe, in order to understand not only the *construction* of the several *lines*, but likewise the principle of the Sector itself, that as the lines on it recede from each other, as they proceed from the centre, so the distance between
any

any two figures of the same denomination in any pair of lines, must always bear an *exact proportion* to their distance from the centre : for example, let AB, AC, fig. 4, represent any two lines on the Sector, it is evident that as the part A 2, on either of the lines, is twice as long as A 1, so is the dotted line drawn from 2, or one line to 2 on the other, twice the length of that from 1 to 1 ; and, for the same reason, the dotted line from 3 to 3, is three times as long as that from 1 to 1, because the two points from whence it is drawn are so many times as distant from the centre A. So that the *proportions* will be always the same, whether they are taken laterally or parallel-wise ; and therefore, though the *lateral* scales of chords, sines, tangents, &c. are of one determined length, being all (except the secants) to one radius, yet the *parallel* scales are variable, and may be had to different radiuses ; and herein consists the excellence of the Sector beyond that of the Plain Scale, as we shall see in the following examples.

As the terms *lateral* and *parallel* will frequently occur, if the reader is not already apprized of their meaning, it will be necessary to inform him, that the term *lateral* is applied to any distance taken *lengthwise* of the Sector ; that is, with *both* points of the Compasses on the same leg ; and that the term *parallel* is applied to any distance taken *cross-wise*, or with one point of the Compasses on each leg ; of course the distances A 1, A 2, A 3, are *lateral distances*, but those of 1, 1. 2, 2. 3, 3. are *parallel distances*.

USE OF THE LINES OF LINES, OR EQUAL PARTS.

1st. To divide a given line into any number of equal parts, suppose 7, take the given line in your Compasses, and setting one foot in a division of equal parts that may be divided by 7 (as 70, for example), open the Sector till the other point falls on 70 in the other leg; or, in other words, make the length of the proposed line a *parallel* distance from 70 to 70, on the Lines of Lines marked L, and then while the Sector remains in that position, take with the Compasses the parallel distance from 10 to 10, and (as 10 is the 7th part of 70) it will give the 7th part of the line to be divided.

2dly. The Lines of Lines are likewise used in finding numerical proportions; thus, suppose the 3 numbers 10, 15, and 20 were given, and it were required to find what is called a 4th proportional, that is a number which shall bear the same proportion to the 3d that the 2d does to the 1st, take the 2d term or number *lateral*, by placing one foot of the Compasses at the beginning of the line, and extending the other to 15; then apply the points so opened parallel-wise from 10 to 10 (the first number), by a proper opening of the Sector; then, while the Sector remains thus opened, take with the Compasses the parallel distance from 20 to 20 (the 3d number), and it will reach laterally from the centre to 30, which is the 4th proportional required, for 30 is to 20 as 15 to 10. It is necessary to observe here, that the numbers 1, 2, 3, &c. properly stand for 10, 20, 30, &c.

&c. as there are 10 divisions between each of them, but they may likewise be taken for 1, 2, 3, &c. or 100, 200, 300, or for thousands, as occasion may require: thus, suppose it were required to find a 4th proportional to the numbers 8, 32, 70, take as before the 2d number lateral (reckoning by divisions), then apply this parallel to the first number. But as the Sector will not admit of being opened wide enough to make a parallel at the 8th division, we must now count by figures, and take the parallel from 8 to 8, which, instead of standing for 80, must now only stand for 8, the decimal or 10th part of 80; then take the parallel of the 3d number at the figure 7, which now stands for 70 as usual, and that distance will reach laterally to the 28th division; but as 10 times the value of the first term or number was taken by reckoning 80 only as 8, so the amount of the 4th term must be multiplied by 10, which will make 280, the number required.

3dly. As all questions in multiplication and division are reducible to *proportions*, we may, if occasion require, both multiply and divide by the Sector: thus, to multiply by the lines of equal parts, take the lateral distance from the beginning of the line to the given multiplicator, make it a parallel distance at 1 and 1, or 10 and 10, and keeping the Sector in that disposition, take the parallel distance of the multiplicand, which measured laterally will give the product: thus, suppose it be required to find the product of 8 multiplied by 4, take the lateral distance from the centre to 4, then open the Sector till you fit this lateral distance to

the parallel of 1 and 1, or 10 and 10; then take the parallel distance of 8 (the multiplicand), which will measure laterally to 32, the product required.

4thly. To divide by the line of equal parts, extend the Compasses laterally from the beginning of the line to 1, and open the Sector till you fit that extent to the parallel of the divisor, then take the parallel distance of the dividend, which, measured laterally, will give the quotient. Suppose, for example, you want to divide 36 by 4, extend the Compasses laterally from the beginning of the line to 1, and open the Sector till you fit that extent to the parallel of 4 (the divisor), then with the same opening of the Sector take the parallel of 36 (the dividend), and that extent, measured laterally, will reach to 9, the quotient required.

As there are two or more lines on every different scale, between which the divisions are drawn, it may be proper to observe, that in using the Sector the inside lines *only* are to be taken, as they *only* are in a direct line with the centre of the instrument.

USE OF THE LINES OF CHORDS.

1st. To draw two lines, making with each other any given angle, suppose 30 degrees, having drawn one line, as BA, fig. 2, open the Sector to any degree you please, and taking the parallel distance from 60 to 60 on the Lines of Chords marked C, with that extent of the Compasses, place one foot on the point where the lines are to meet as at B, and with the
other

other describe part of a circle, as ab , reaching from the line first drawn as far as where you guess the other line will fall, or farther; then with the same opening of the Sector take the *parallel* distance from 30 to 30 on the Lines of Chords, and placing one foot of the Compasses at b , where the circle touches the line AB , make a mark at that part of the circle where the other foot falls, which suppose at a , and drawing a line from B through that point, the two lines AB , and BD , will make the angle required.

2dly, Suppose the lines already drawn, and you want to find by the Sector what angle they make with each other, open the Sector as before to any degree you please, and take the *parallel* distance from 60 to 60 with the Compasses; then placing one foot on the point B where the two lines meet, make a mark on each of the lines where the other foot falls, which suppose at a and b ; take the distance between the points a b , and applying it *parallel* on the Sector, remaining open as before, it will reach from 30 to 30; and thus you have the quantity of the angle as desired.

3dly, But as the two legs of the Sector may be opened to any angle whatever, we have hence a very simple and expeditious method of measuring or laying down any angle by the Sector, which in this case may be made to answer the double purpose of a Protractor and Ruler. But here it must be observed, that every pair of lines on the Sector (and that of the Chords among the rest) make an angle of 5 degrees on the 6-inch Sector when *shut*, and therefore in laying down

an angle by the sides or edges of the Sector, you must open the Lines of Chords 5 degrees more than the angle intended to be drawn, and in *measuring* the same number of degrees must be *deducted* from the extent given by the Sector. Thus to lay down an angle, suppose of 30 degrees, to the 30 degrees intended to be laid down add 5, which will make 35; extend the Compasses laterally from the centre to 35 on the Line of Chords, and open the Sector till the points of the Compass so opened reach from 60 to 60; and the sides of the Sector being then opened to 30 degrees, any lines drawn with the pen or pencil by them will make the required angle; in doing which you may make use of the two outsides, or the two insides of the Sector, or one of each, as suits your purpose.

To *measure* an angle by the sides of the Sector, open them till any two of the edges correspond with the two lines forming the angle, then with that opening take the *parallel* distance from 60 to 60, and it will give, if measured laterally, just 5 degrees more than the measure of the angle.

It is proper to mention that the angle made by the sectoral lines when the Sector is shut, is not *always* exactly 5 degrees, but is sometimes *more* and sometime *less*; in order therefore to know how much to add or deduct in using the Sector as above directed, it will be necessary, in the first place, to take the distance from 60 to 60 on the Lines of Chords (the Sector being shut), and the distance measured *laterally* on either of the same lines will give the quantity of the angle to be added or deducted in so using it.

USE

USE OF THE LINES OF SINES, TANGENTS, AND SECANTS.

We have already observed, that by the very nature of the Sector we can obtain a parallel to any radius not exceeding the whole length of the Sector when opened, so that, having a *radius* given, we easily find the Sine, Chord, &c. belonging to it: thus, if the chord, sine, or tangent of 30 degrees, to a radius of one inch and half were required, the reader will recollect that the chord of 60 degrees, the tangent of 45 degrees, and the sine of 90 degrees, are all of them severally equal to the radius of the circle from whence they are taken; and that as *every* pair of lines contains the same angle (viz. 5 degrees) when the Sector is shut, to whatever angle any one of them is opened to the same are all the rest opened; hence you have only to open the Sector till the distance between 60 and 60 on the lines of chords is equal to an inch and a half, and all the lines and divisions of chords, tangents, and sines on the Sector will correspond with that radius; therefore if, with the Sector thus opened, you take the parallel distance between 30 and 30 on the lines of tangents, and apply it to fig. 6, where the radius is 1 inch and a half, it will give the length of a tangent of 30 degrees. If you take in like manner the parallel of 30 on the lines of sines, it will give the sine of 30 degrees to the same radius; and so of the rest. The lines of secants however, marked SE. and likewise the lesser tangents marked TA. though they contain the same angle with all the other lines, do not correspond with

with them as to the radius, being taken from a shorter one, viz. one of 1 inch and half (instead of 6 inches, as the rest are), and therefore they both begin at that distance from the centre of the Sector. In using these two lines the radius is to be taken from the beginning and not from the end, as in the lines of sines, chords, and larger tangents; suppose therefore it were required to find the tangent of 60 degrees to a radius of an inch and half, in this case as the *larger* tangents, or those marked T. end at 45 degrees, the lines of *lesser* tangents, marked T A. must be used; and therefore, taking an inch and half in the Compasses, open the Sector till the two points fall on the beginning of both the lines of lesser tangents; then take the *parallel* distance from 60 degrees to 60 degrees in the same line, and it will give the tangent required, as will be seen by applying the Compasses to fig. 6, where they will measure 60 degrees on the line of tangents.

If it were required to find the secant of 60 degrees to a radius of 1 inch and half, make the radius a parallel distance at the beginning of the 2 lines of secants marked S E. and taking the parallel of 60 and 60 on the same line, it will give the length of the secant required.

But the above lines are not always used singly; the lines of sines, tangents, and secants, are used in conjunction with the lines of lines in solving the different problems of plain trigonometry; thus, in the triangle A, B, C, fig. 3, which is right angled at C, if the side BC be given equal to 260, and the angle at B equal to 33 degrees and 35 minutes, it is required to
find

find the side AC. Here, if we suppose the line BA to be the radius, AC will be the sine of the angle A, B, C, or if we make BC radius, the line CA will be the tangent; suppose then BA to be the radius, and CA a sine, to find it you use the lines of sines in conjunction with the lines of lines as follows: take the lateral distance of 260 on one of the lines of lines (reckoning the figures 1, 2, 3, as hundreds, and consequently every division as 10, for otherwise they will not extend so far as 260); then as the angle at B is 33 degrees 35 minutes, that at A *must* be 56 degrees 25 minutes, for in every *right-angled* triangle the two lesser angles taken together are equal to one right angle or 90 degrees; or in other words, in every triangle whatever the sum of the 3 angles taken together is always 180 degrees; make therefore the said lateral distance of 260 a parallel distance at 56 degrees 25 minutes on the lines of sines, then the parallel distance of 33 degrees 35 minutes will measure laterally (reckoning as before) 173 on the lines of lines for the length of AC.

But if, instead of BA, we make BC radius, then the line CA is a tangent, and its length may be found by using the tangent lines in conjunction with the lines of lines: thus, take the distance 260 as before, and make it a parallel distance at the tangent radius, viz. from 45 degrees to 45 degrees, then with the same opening of the Sector take the parallel of 33 degrees 35 minutes, on the same lines of tangents, and that distance will measure laterally on the lines of lines 173, which is the length of the line AC.

In

In like manner the side CA, and the two angles at B and A being given, you find the length of the line BC, which is either the *sine* or *tangent* of the angle B, C, A, according as you take either AB or CA for the radius; let AB be considered as radius, in that case the line BC is a *sine*, and you find it thus: take the line AC equal to 173 laterally, on the line of lines, and make it a parallel distance at 33 degrees 35 minutes on the lines of sines, then the parallel of 56 degrees 25 minutes, on the same lines, will give 260 for the line BA.

Lastly, to find the Hypothenuse or longest side of the triangle ABC (the base BC, and the angles at B and A being given); here, if we consider BC as the radius, then the line BA is the secant of the angle B; and to find it use the lines of secants in conjunction with the lines of lines: thus, take the distance of 260 laterally on the lines of lines, and make it a parallel distance at the beginning of the lines of secants, and the parallel of 33 degrees 35 minutes on the same lines, will give 320 on the lines of lines for the side BA.

USE OF THE LINES OF POLYGONS.

The last of the *sectoral* lines we have to notice are the lines of Polygons marked POL. The word Polygon signifies, in general, a figure of many equal sides, and they are *particularly* called *Pentagons*, *Hexagons*, &c. according to the *number* of their sides; and as the angles formed by the sides are all of them equidistant

distant from one common centre, they are therefore bounded by, or described within, a circle. We have before observed that the chord of 60 degrees is equal to the radius or semi-diameter of the circle. When, therefore, you are to describe a Polygon containing any number of sides, from 4 to 12, you are to open the Sector till the parallel distance between 6 and 6 on the line of Polygons is equal to the radius of the circle, within which the Polygon is to be inscribed, and then taking the parallel distance between the two figures, which answer to the number of sides the Polygon is to have, it will give the length of each side: thus, suppose you were to describe an Octagon, open the Sector till the parallel distance between 6 and 6 is equal to the radius of the circle within which the Polygon is to be inscribed, and then the parallel distance between 8 and 8 on the same lines will be the length of each side of the Octagon.

These are *all* the lines called *Sectoral*, or that are to be used Sector-wise; but, beside these, there are the lines called *Gunter's*, which have been already mentioned, which are three in number, viz. the line of numbers, the line of sines, and the line of tangents; they extend nearly the whole length of the Sector, which when they are used is to be opened to its full extent like a common rule; these are called *artificial* lines, because they are only the logarithms, or proportions of the natural numbers, sines, and tangents, laid on lines of scales: the manner of *constructing* them cannot be easily explained to those who are unacquainted with the nature of logarithms, and will be easily comprehended

hended by those who are; but the manner of using them will admit of an easy explanation, and they will in a very simple manner give answers to the several questions in the foregoing examples, as well as many others. Mr. Martin, in explaining the principles on which they are used, observes that "Logarithms are only the ratios of *numbers*, and the ratios of all proportional numbers are equal; and all questions in Multiplication, Division, the Rule of Three, Trigonometry, &c. are stated in proportional terms or numbers;" hence the general method of using them is as follows:

Suppose it were required, as before, to find a 4th proportional to the numbers 10, 15, 20, the method of finding it by the Sectoral lines of numbers has been already explained; to find it by Gunter's line of numbers, marked NUM. open the Compasses till the points take in the distance between the 1st and 2d numbers or terms, and then the same opening of the Compasses will reach from the 3d term or number to the 4th. Thus, for example, in the numbers given, 10, 15, 20: if you place one foot on 10, and extend the other to 15 in the line of numbers, and then with the same extent place one foot on 20, the other will reach to 30, the number required. It may here be necessary to observe, that on the line of numbers the figures begin at 1 and are continued to 10; after which they begin at 1 again, and go on as before at the same distance as in the first row. If the figures in the first row are considered as units, those in the second must stand for tens; or, if those in the first stand

stand for tens, those in the second will be hundreds, and so on; and hence it is obvious why the distances of the several figures, &c. in both rows, answer to each other, there being the same proportion between 2 and 3, for instance, as between 20 and 30, or 200 and 300, &c. so that if it were required (as before in the examples of Sectoral lines) to find a 4th proportional to the numbers 8, 32, 70, extend the Compasses from 8 to 32, when, on placing one foot of the Compasses on 70, in order to find the 4th term or number, you will find the other reach beyond the end of the Sector; let therefore the figure of 7 in the first row be called 70, and placing one foot of the Compasses on it, the other will reach to 28; but as 7 was called 70, so 28 must now be called 280, which is the 4th proportional required, for 280 is to 70 as 32 to 8. Gunter's lines of sines and tangents may likewise be used in conjunction with the line of numbers, and the same problems solved by them as by the *Sectoral* lines of numbers, sines, and tangents, before treated of. For example: let it be required to find the line AC, fig. 3, the line BC and the angle at B being given as before, take the distance from the radius 45, or end of the tangent line marked TAN. to 33 degrees 35 minutes on the same line, and then placing one foot of the Compasses at 260 on the line of numbers, marked NUM. the other will reach to 173 for the line CA. Here we have considered CA as a tangent, but if BA were the radius, and CA consequently a sine, its length may be found by the line of sines; in conjunction with that of the numbers: thus,

take

take on the line of sines, SIN. the extent from 56 degrees 25 minutes, to 33 degrees 35 minutes (the angle at B), and then placing one foot of the Compasses at 260, on the line of numbers, the other will fall on 173 as before. In this manner may the several parts of the triangle be found, even in a more ready and simple manner than by the use of the sectoral lines.

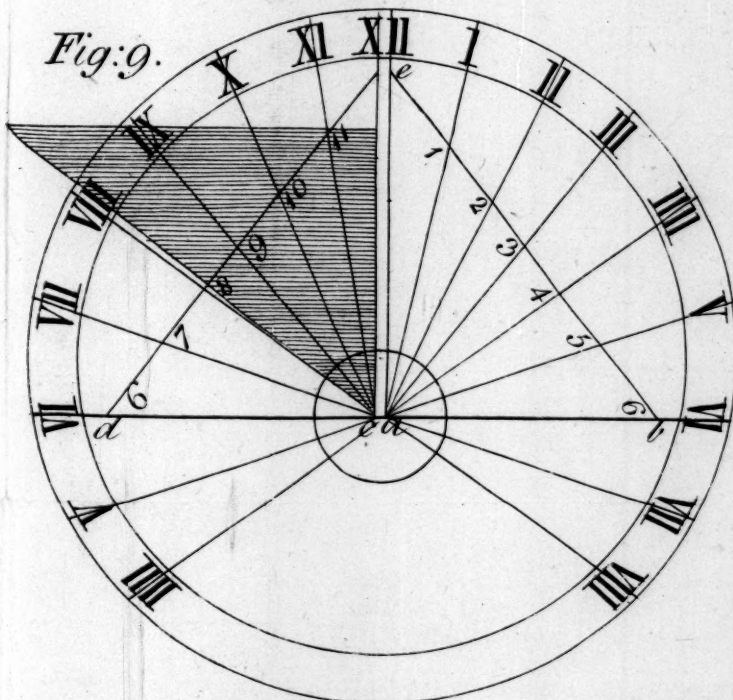
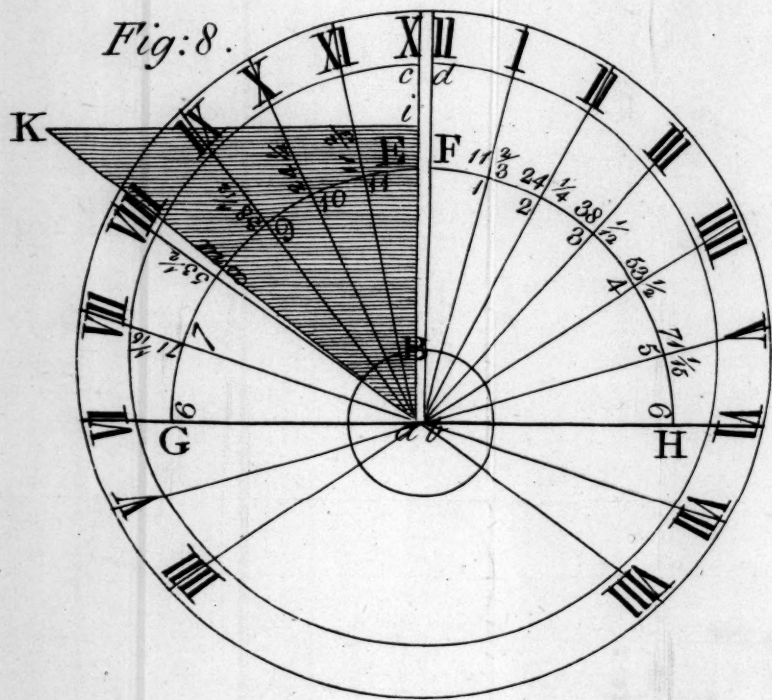
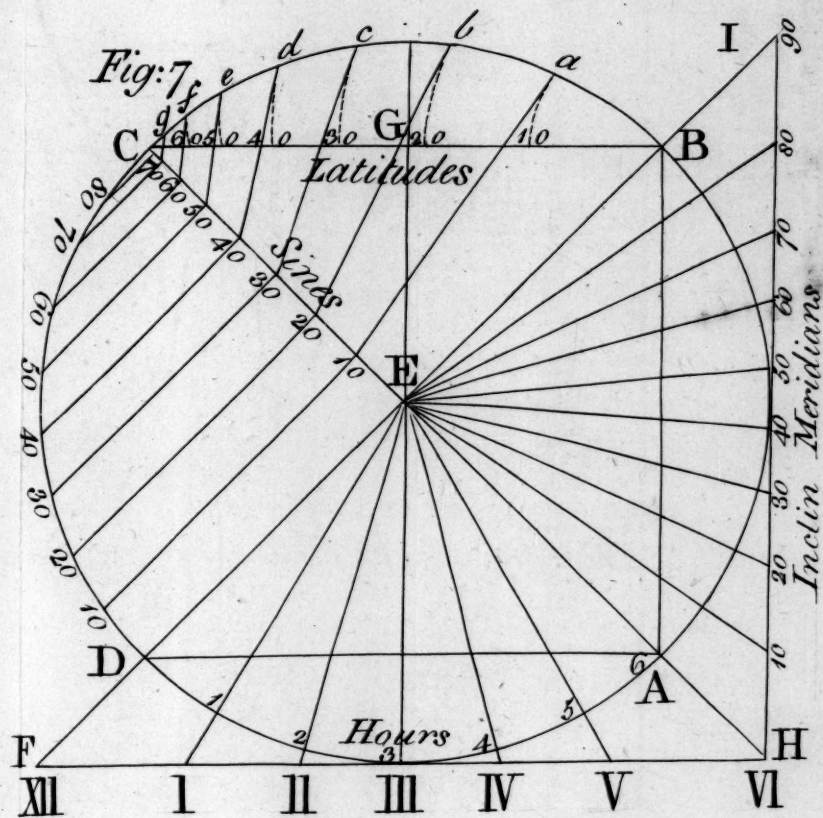
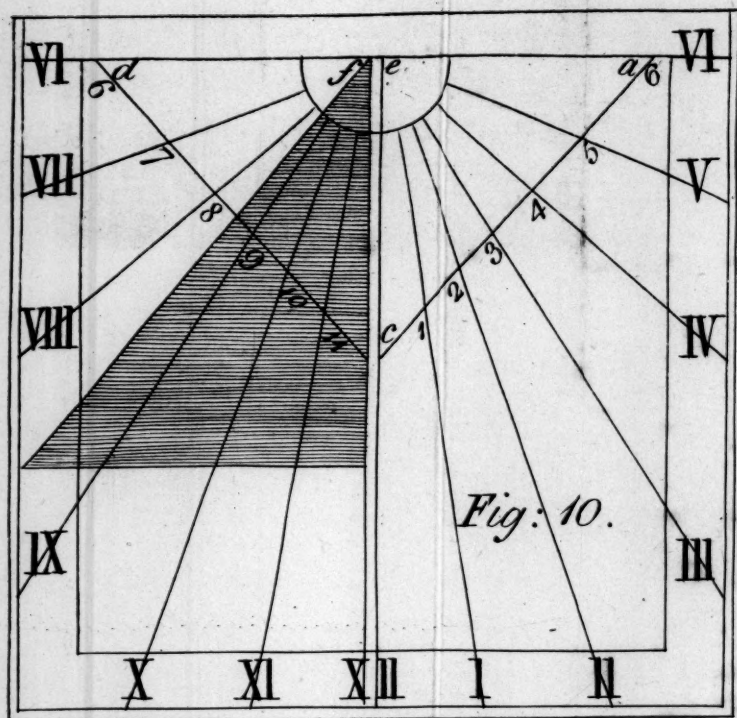
OF THE DIALLING LINES ON THE SECTOR, viz.

THE LINE OF INCLINATIONS OF MERIDIANS, LINE OF
CHORDS, LINE OF LATITUDES, AND LINE OF HOURS.

These lines, as well as those last mentioned (viz. Gunter's), are placed *singly* on the Sector, but are not extended like them from one leg to the other, being the shortest lines of any on the Sector, as the two longest only reach half the length of the leg; they are all placed on the contrary side to the Gunter's lines, and are marked IM. or IN. MER. for Inclinations of Meridians; Cho. for Chords, Lat. for Latitudes, and Hou. for Hours: we have already observed that they are not on every Sector (as all the rest are), but only on the better sort, or those which are termed *full divided*.

That the use of these lines may be more easily understood, it will be necessary to shew how they are constructed. In order to this, describe a circle A, B, C, D, fig. 7, and divide it into four equal parts by the lines CEA and BED; then divide the Quadrant

CD



CD into 90 equal parts, and draw lines from every division to the line CE. These lines (which must be drawn parallel to DE and perpendicular to CE) will divide CE into the line of *sines* as before explained. The *sines* are of themselves of no use in Dialling, but they are of use in drawing the line of *latitudes*; for if you lay a ruler on the point D, and over the several divisions on the line of *sines*, draw lines till they cut the Quadrant CB, in *a b c d*, &c. and they will divide it into 90 *unequal* parts. Draw now the line CGB, and placing one foot of the Compasses in B, transfer the several lengths or distances B *a*, B *b*, B *c*, &c. to the line CGB, and it will be a line of Latitudes, which must be numbered from B. Note, though this line is *divided* into 90 degrees, yet both in the figure and on the Sector it is only *numbered* to 60 or 70, the figures 80 and 90 being omitted for want of room.

Draw the line FH, with its extremities, equally distant from the centre of the circle E, and, touching the circle in the middle at the figure 3, divide the Quadrant AD into six *equal* parts, and number them 1, 2, 3, &c. and through these several divisions draw lines from the centre of the circle to the line FH, and it will be a line of six hours, which by the same rule may afterwards be divided into quarters, &c.

Next draw the line IH similar to FH; divide the Quadrant BA into 90 *equal* parts, and draw lines from the centre of the circle through the several divisions in the Quadrant to the line IH, and it will be a line of inclinations of the meridian. As to the line of

D

chords,

chords, it is formed as before directed, page 22, and therefore need not be again described.

From considering the figure, the reader will readily perceive that the line of latitudes and that of chords, each being of 90 degrees, and contained within a Quadrant of the same circle, *must* be of the same *length* (as is the case on the Sector), but the divisions of the former are much more *unequal*, the distances *decreasing* as the numbers increase much more sensibly than on the line of chords. It is likewise evident why the line of *hours* and the line of *inclinations* of *meridians* are of the same length on the Sector, and why the divisions on both of them are closer at the middle and regularly widen toward each end, differing only in the number of their divisions; and hence, if every hour were divided into 15 parts, the line of hours would become a line of inclinations of meridians, which in practice would perhaps be less embarrassing than the present mode of making two distinct lines of them.

In treating of the Dialling lines on the Sector, it will not be expected that we should describe the method of making *all* the various kinds of dials to which they may be applicable; this would be to form a Treatise on Dialling rather than a description of the Dialling *lines*, and would be of but little use when done, as there are several treatises on that subject already extant, which those who wish to become *complete* masters of the *whole* art of Dialling will do well to consult: all that we mean in this place is to give a few examples of the use of these lines as laid down on the
Sector,

sector, in the construction of some of the plainest as well as most useful kinds of dials; and, first,

OF THE USE OF THE LINE OF CHORDS.

Suppose it were required to make a horizontal dial, such as represented in fig. 8. From the centre B describe the two outside circles between which the hours are afterwards to be drawn; draw the line GH for the equinoxial or six o'clock line, dividing the circles with two unequal parts. Note, the reason why the line is not drawn through the centre B is, that the hours about noon would fall too near each other, but by taking the six o'clock line behind the centre of the dial, the figures about twelve are farther asunder, as will easily be seen by the figure. From *a b*, the middle of the line GH through B, the centre of the two outside circles, draw the double line *a b c d*: this double line is the twelve o'clock line, and is perpendicular to the six o'clock line GH; it is called the *substile*, because the gnomon or stile is placed *over* it, and must be equal in breadth with the thickness of the stile itself. This being done, the hours may be drawn by the line of chords as follows: open the Compasses to 60 degrees on the line of chords, and with that opening as a radius describe the Quadrant FH, placing one foot of the Compasses on the point where the substile and equinoxial lines meet (as at *b*;) with the same opening describe likewise the other Quadrant EG, placing one foot on the point *a*: then take from the same line of chords the several distances

of 11 degrees two thirds for the distance of the hour 1 from 12; 24 degrees 1 quarter for the hour 2; 38 degrees 1 twelfth for the hour 3; 53 degrees 1 half for the hour 4; and 71 degrees 1 fifteenth for the hour 5; and mark them on the Quadrant FH (measuring from the 12 o'clock line, and numbering them 1, 2, 3, &c.); then, laying a Ruler on the point *b*, over the several divisions on the Quadrant, through them draw lines, as in the figure; and where they fall on the outside circles, there place the figures for the afternoon hours. The very same process on the Quadrant EG will give the *morning* hours, taking the distance of 11 degrees two thirds for the hour of 11; 24 degrees 1 quarter for 10; and so on: after which continue the lines of 4 and 5 in the afternoon through the centre to the opposite part of the circle, and you will have the places of 4 and 5 in the morning; and, continuing in like manner the lines from 7 and 8 in the morning, you will have the like hours for the afternoon on the opposite side of the Dial.

The line of chords is likewise useful in finding the elevation of the Gnomon, or Stile: thus,

Having described, as before, the Quadrant EG with the chord of 60 degrees as radius, take the latitude of the place from the same line of chords, which, if for London, will be 51 degrees 32 minutes, and, marking it down on the Quadrant from the Substile, as at *m*, draw the line *amK*, and cross it by the line *iK*, perpendicular to the Substile, and you have the exact form of

of the Stile, or the angle which its upper edge will make with the plate; let this be fixed perpendicularly on the plate, and the Dial is finished.

USE OF THE LINES OF HOURS AND LATITUDES.

Instead of using the line of chords for drawing the hour lines, they may be drawn by means of the lines of latitudes and hours: thus, having drawn the double meridian, or substile line, and crossed it at right angles by the equinoxial, or six o'clock line, as before, from the scale of latitudes marked Lat. take the latitude of the place for which the Dial is intended (as 51 degrees 32 minutes for London), and with the Compasses set that extent from *a* to *b*, and from *c* to *d*, on the 6 o'clock line, as in fig. 9; then take the whole length of the line of 6 hours, marked Hou. in the Compasses, and, putting one foot on *b*, let the other fall where it will on the line *ea*; make a mark where it falls (which suppose at *e*), and from this mark to the point *b* draw the line *eb*; then, extending the Compasses from the beginning of the line of hours to 1, mark this distance from *e* toward *b*; do the same for all the other hours, and, having marked where they fall on the line *eb*, draw lines from the point *a* through each of these marks to the circumference of the dial, and on these lines place the hours, as in the figure; and, having drawn the afternoon hours, a similar process on the other side of 12, or the substile line, will

give the forenoon hours, making the distance between 12 and 11, on the one side, the same as between 12 and 1, on the other; from 11 to 10 the same as from 1 to 2, and so on. When this is done you have only to continue the lines from 4 and 5, and from 7 and 8, through the centre, as before, and take from the line of chords the height of the subtile equal to the latitude, as before directed, and the Dial is completed.

From what has been said, any person will be able to construct what is called a *Horizontal Dial*, such as is placed on a pedestal in gardens, or on the outside of windows, or in short any Dial, of whatever form, which is placed flat, or on a level with the horizon. But besides these there are what are called *erect* or *vertical* Dials, such as those which are described on, or placed against, an upright wall: for these a wall looking *toward* the south should be chosen; and, if the wall be *directly* south, the process for making it is the same as for the horizontal Dial, only the figures for the hours must be reversed, as in fig. 10, and, instead of taking the *latitude* of the place for the distances *ea*, and *fd*, and for the height of the stile (as directed for the horizontal Dial), you are now to take what is called the *complement*, or *co-latitude*, that is, so many degrees as are wanting to make the latitude equal to 90 degrees: thus, at London the co-latitude, or complement of the latitude, is 38 degrees 28 minutes; because 38 degrees 28 minutes added to the latitude, 51 degrees 32 minutes, make 90 degrees; and therefore 38 degrees 28 minutes are to be taken from the line of latitudes for the distance

distance from *e* to *a*, and from *f* to *d*, and likewise from the line of chords for the height of the style.

Note, as the sun cannot shine longer than 12 hours on erect Dials, there is no occasion, as on the horizontal Dial, to extend the hour figures farther than from 6 to 6.

THE USE OF THE LINE OF INCLINATION OF MERIDIANS.

But if, instead of being *directly* facing the south, the wall, on which your Dial is to be drawn or fixed, inclines toward the east or west, the Dial is then called a *Declining* Dial, and the substile, instead of being on the meridian, or 12 o'clock line, will make an angle with it, greater or less according as the face of the Dial, or the wall against which it is placed, inclines more or less from the south toward the east or west. This *difference* between the substile and 12 o'clock line is called in Dialling the *inclination of meridian*. If the wall declines *eastward*, the substilar line will fall among the *morning* hours; but, if it declines *westward*, it will fall among the *afternoon* hours. Now, in order to know at what place among the hours the substilar line will fall, take from the line of inclinations of meridians, marked IM. just so many degrees as are equal to the inclination of the wall either east or west (which we suppose to be already known, or, if not, it may be found by the method hereafter described), and apply the distance so taken between the points of the Com-

passes to the line of hours on the Sector, and it will shew where the substilar line will fall: thus, if the wall's inclination be 18 degrees 30 minutes west, place one foot of the Compasses at the beginning of the line of inclinations of meridians, and extend the other to 18 degrees 30 minutes on the same line; then the same extent of the Compasses will reach from the beginning of the line of hours to a quarter past 1 on that line; and therefore the substilar line must fall at a quarter past 1 on the Dial; or if the Dial declines as much eastward, then, by the same rule, the substile must be drawn at three quarters past 10, or one quarter before 11; and thus, by the line of inclinations of meridians may always be found the place of the substile line, when the declination of the wall is known; in every other respect a declining Dial is made in the very same manner as the direct south Dial.

It may be necessary here to add that, though upright Dials are generally of a *square*, and horizontal ones of a *circular*, form; yet neither is of any consequence as to the construction; they may both be made of either form indifferently.

It is likewise easy to conceive that the lines of chords, hours, latitudes, and inclination of meridians, usually placed on the Sector, will serve for Dials of different sizes, or indeed for any size that is large enough for the line of 6 hours to fall within it; but if a Dial should be wanted too *small* for the lines on the Sector, or if, for the sake of greater accuracy in large Dials, the reader should wish to make use of larger scales than those on the

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the Sector, he may easily, by attending to the directions already given, construct them himself of any size he pleases.

Though the method for finding the true meridian is not strictly connected with our primary intention, yet, as it is of the utmost consequence, both in *fixing* horizontal Dials as well as in *describing* vertical ones, it may perhaps be acceptable to those who may wish to amuse themselves, either in making or fixing Dials, to describe the most simple method of finding the true *south* point.

Choose a board smoothly planed, and paste white paper on it; then make a hole in the centre to contain an upright pin, and from it, as a centre, draw several circles: or it would be still better if, when the pin (which must be pointed at the top) is fixed firmly in its place, you take a ruler, or any other piece of wood, with a point at one end of it, and laying it at different parts of its length on the top of the pin, draw with the point at the end the several circles on the board; by this means the top of the pin will be *exactly* in the centre of all the circles. This done, early in the morning place the board thus prepared exactly level in the place where you wish to make the observation (the best time to do which is about the 10th of June, when the sun is near the solstice), and wait till the shadow of the top of the pin touches any one of the *circles*, and there make a mark; come again as much *after* 12 at noon as this observation was made *before* that hour, and make a mark at that place
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on the *same* circle where the shadow of the pin's top *than* falls; divide the distance between these two marks very accurately, and the middle will be the *meridian* or true *south*; but, for fear the sun should not happen to *shine* exactly at the time you want it in the afternoon, it is best to mark the places in the morning, where the shadow touches *different* circles, which it will do at different times, going from the outer to the inner circles, as the sun rises: that you may take whichever happens to suit for the afternoon observation. There is, however, a *very small* error in this method, which you may correct if you think proper, which arises from the sun's *proper* motion, by which he loses as much in six hours as he gains in one minute of an hour by his *daily* motion. To be *perfectly* correct, therefore, you must add as much to the *last* or *afternoon* mark, as the shadow moves in one minute, which is easily found by a watch or clock.

There are two kinds of Instruments, viz. the Proportional Compasses and Marquoi's Parallel Scales; which, though they do not make a part of every good Case of Instruments, are yet sometimes found in them, or used in conjunction with them: the former is an old invention compared with the latter, which have not been long introduced, though possessed of peculiar advantages above the common Parallel Ruler. To those who are, or may be, in possession of them, and are not acquainted with their use, a brief description of them will not, it is presumed, be unacceptable.

The Proportional Compasses consist of two sides, each of which has a short steel point at one end, and a much longer one at the other; the two sides are exactly the same length, and are made so like each other in every respect as to seem like one piece when together, before they are opened. They have each a long *groove* cut through them, and through this groove runs a steel pin with a brass nut at each end, by means of which it is made to slide along the grooves, and may be fixed in any part of them: this pin, when fixed by the nut in any part of the groove, serves as a centre for the two sides to turn upon; by this means the two points at each end of the Compasses may be opened in any proportion to each other: if, for instance, the centre about which they turn is placed in the middle, it is plain that the opening at both ends of the Compasses will be the same; but, if the centre is fixed *nearer* to the end where the short points are, the long points of the Compasses will open in the *same proportion* wider than the short ones, as the centre is more distant from them, than it is from the short ones; and hence they become *Proportional Compasses*.

There is a small piece of brass fixed to the sliding centre, across which is drawn a fine line, and on each side of the groove in which it slides are other lines, or divisions, to which it is to be set in using, and by which the several proportions are determined. The lines, or scales, on the sides of the Compasses are a line of *circles*, a line of *lines*, and sometimes lines of *plans* and *solids*.

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The line of Circles (so called) is more properly a line of Polygons, as its use is to inscribe polygons of any number of sides, from 6 to 20 in a circle: its use is very easy, as you have only to move the centre till the line across the sliding piece corresponds with the figure which expresses the number of sides the polygon is to have, and fixing it there, if you open the Compasses till the distance between the longest points is equal to the radius of the circle, in which the polygon is to be inscribed, the shortest point will give the length of one of its sides; thus, if an octagon were wanted, slide the line on the sliding piece to the line at fig. 8, on the line of circles, and fixing it there, open your Compasses, till the distance between the long points is equal to the radius of the circle which is to contain the octagon, and the short points will give the length of one side of the octagon; and so of all the rest. It is almost needless to mention, that by this means a circle may be divided into any number of equal parts from 6 to 20, and 1, 2, 3, or more of those parts taken at pleasure, when a polygon is not intended to be drawn.

The use of the line of lines is still more simple; by it the points at each end may be opened in any proportion to each other: thus, if you slide the index to 3, the long points will be at any opening of the Compasses, three times as distant as the short ones; if to 4, they will be four times, and so on: and thus a line be divided into any number of equal parts, or lines drawn or measured in any proportion to each other,
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all which is so plain that no examples can be necessary.

In the same manner are the lines of plans and solids to be used when they are found on the Proportional Compasses, as plans and solids bear the same proportion to each other with lines.

All the operations performed with the Proportional Compasses may likewise be performed by means of the Sector; but their advantage is, that those operations, which, when performed by the Sector, require two instruments (viz. the Sector and a pair of Compasses or Dividers), may be performed by the Proportional Compasses only, so far as they are applicable.

Marquoi's Parallel Scales consist of two straight rules commonly of one foot long (but sometimes they are made much longer), and a flat piece in the form of a right-angled triangle, whose longest side is three times the length of the shortest. They are made either of box or ivory, and their use to draw either parallel lines or perpendiculars; and with this peculiar advantage, that any number of either may be drawn at any required distance, even to the sixtieth of an inch, and that in the most simple, accurate, and expeditious manner.

For this purpose there are eight scales, viz. two on each side of the rulers: these consist of equal parts of an inch from 20 to 60, and are divided in the same manner as those on the plain scale before described. Each of these is placed under an *artificial* scale, which

which artificial scale is on the very edge of the ruler, and answers to the natural scale underneath it, in the proportion of three to one; that is, the divisions and subdivisions on it are three times as distant from each other as on the natural scale, and they are numbered both ways from the centre, where there is an O.

In the centre of the longest side of the triangular piece there is a star or flower-de-luce, from which there is a line drawn which serves as an index in using it.

To use these Scales as a parallel ruler, place the *longest* side of the triangular piece against the edge of one of the rulers, with the *shortest* side toward the right hand, and move them *together* till the sloped edge of the triangle is in the place where the first line is to be drawn; or, if a line is already drawn to which one or more parallels are wanted, it must be moved till the said edge corresponds with that line, and then any number of parallels may be drawn, either above or below the line, by sliding the triangle either to the right or left along the ruler, taking care to keep the ruler steady without moving, and pressing the triangle close to the edge of it.

By means of the several Scales on the edges of the ruler, parallels may be drawn at any required distance: suppose, for example, any number of parallels were wanted to be drawn at the distance of the thirtieth of an inch from each other, choose the scale marked 30, and placing the triangle against it (as before directed) with the star or flower-de-luce on the triangle,

angle, to correspond with the O in the middle of the artificial scale on the edge of the ruler, if you move the triangle one division, and then draw a line, it will be one thirtieth of an inch either above or below the given line, according as the triangle was moved to the right or left. If you move the triangle *ten* of the divisions, then the lines will be one third of an inch distant; if 30, then the lines will be one inch distant; and thus by using different scales, or moving the triangle more or less along the edge of the rulers, parallels may be easily drawn at any distance from each other, or from any given line.

It is easy, with a very little care, to move the triangle exactly from one division to another; but one peculiar excellence of these scales is, that, as the artificial scale, which is the guide in moving the triangle, is three times as long as the natural scale, if any trifling error should happen in placing the triangle to the several divisions, the error in drawing the lines is only one third of what it would be if the two scales exactly corresponded.

Another advantage of these scales above the common parallel ruler is, that perpendiculars may be very expeditiously drawn; for, if you place the edge of one of the rulers to correspond with any given line, then placing the short end of the triangle against that edge, the sloped side of it will be perpendicular to the ruler, and consequently a line drawn along the sloped side of the triangle will be perpendicular to the given line.

T H E E N D.

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CONSIDERATIONS

ON THE

Utility of Conductors for Lightning:

IN WHICH

The Nature and Properties of the Lightning are explained, and the admirable Uses of Conductors in preventing its calamitous Effects clearly pointed out.



THE END